

Formulas for accelerator physics and synchrotron radiation

Curvature radius of ultra relativistic particles in a storage ring:

$$\frac{1}{r} = e \cdot \frac{B}{p} = \frac{c \cdot e \cdot B}{c \cdot p} = c \cdot e \cdot \frac{B}{E}$$

$$\frac{1}{r[m]} = 0.299792458 \cdot \frac{B[T]}{E[GeV]}; \quad 1[T] = 1[Vsec/m^2]$$

(*Example* : $r = 12.1849m$; $B = 1.221884T$; $E = 4.5GeV$)

(*Example* : $r = 12.1849m$; $B = 1.450885T$; $E = 5.3GeV$)

(*Example* : $r = 191.7295m$; $B = 0.20877T$; $E = 12GeV$)

(*Example* : $r = 608m$; $B = 0.15142T$; $E = 27.6GeV$)

Quadrupole strength:

$$k = e \cdot \frac{g}{p} = \frac{c \cdot e \cdot g}{c \cdot p} = c \cdot e \cdot \frac{g}{E}$$

$$k[m^{-2}] = 0.299792458 \cdot \frac{g[T/m]}{E[GeV]}; \quad B = g \cdot x$$

(*Example* : $k = 0.6662m^{-2}$; $g = 10T/m$; $E = 4.5GeV$)

Sextupole strength:

$$m = e \cdot \frac{g'}{p} = \frac{c \cdot e \cdot g'}{c \cdot p} = c \cdot e \cdot \frac{g'}{E}$$

$$m[m^{-3}] = 0.299792458 \cdot \frac{g'[T/m^2]}{E[GeV]}; \quad B = \frac{1}{2} \cdot g' \cdot x^2$$

(*Example* : $m = 6.662m^{-3}$; $g' = 100T/m^2$; $E = 4.5GeV$)

Field index of a synchrotron magnet:

$$n = -\frac{r}{B_0} \cdot \frac{\Delta B_z}{\Delta x}$$

(*Example* : $n = -22.26$; $r = 7.65$; $B_0 = 1.003T$; $\Delta B_z/\Delta x = 2.919T/m$)

Focal length of a quadrupole:

$$\frac{1}{f} = \sqrt{k} \cdot \sin(\sqrt{k} \cdot l)$$

$\approx k \cdot l$ for small k and l

(Example : $f = 2.16m$; $k = 0.7m^{-2}$; $l = 0.7m$)

Wiggler parameter k :

$$k = \frac{e}{2\pi \cdot m_e \cdot c} \cdot B_{max} \cdot \lambda$$

$$k = 0.9337287 \cdot B_{max}[T] \cdot \lambda[cm]; \quad 1[kg] = 1[VAssec^3/m^2]$$

(Example : $k = 7.40$; $B_{max} = 0.6T$; $\lambda = 13.2cm$)

(Example : $k = 28.0$; $B_{max} = 1.25T$; $\lambda = 24cm$)

(Example : $k = 1.61$; $B_{max} = 0.55T$; $\lambda = 3.14cm$)

Focal length of a wiggler (vertical plane):

$$\frac{1}{f} = \frac{\tan \epsilon}{r} = \frac{2\pi^2 \cdot N \cdot k^2}{\lambda \cdot \gamma^2}; \quad \gamma = \frac{E}{E_0}$$

$$= \frac{1}{2}c^2 \cdot e^2 \cdot L \cdot \frac{B_{max}^2}{E^2}$$

$$\frac{1}{f[m]} = 0.04493776 \cdot L[m] \cdot \frac{B_{max}[T]^2}{E[GeV]^2}$$

(Example : $f = 593m$; $L = 2.112m$; $B_{max} = 0.6T$; $E = 4.5GeV$)

(Example : $f = 113m$; $L = 4.00m$; $B_{max} = 1.0T$; $E = 4.5GeV$)

Maximal electron deflection angle in a wiggler or undulator:

$$\Delta = \frac{k}{\gamma} = k \cdot \frac{E_0}{E} = \frac{c \cdot e}{2\pi} \cdot \lambda \cdot \frac{B_{max}}{E}$$

$$\Delta = 0.04771345 \cdot \lambda[m] \cdot \frac{B_{max}[T]}{E[GeV]} \text{ for electrons}$$

(Example : $\Delta = 3.18mrad$; $k = 28$; $E = 4.5GeV$)

(Example : $\Delta = 1.50mrad$; $k = 13.2$; $E = 4.5GeV$)

(Example : $\Delta = 3.18mrad$; $\lambda = 0.24m$; $B_{max} = 1.25T$; $E = 4.5GeV$)

Number of ultra relativistic particles in a storage ring:

$$n = \frac{U \cdot I}{c \cdot Q}; \quad I = \frac{n \cdot Q}{T}; \quad c = \frac{U}{T}$$

(Example : $n = 600 \cdot 10^9$; $I = 0.1A$; $U = 288m$; $Q = e$)

(Example : $n = 6.02 \cdot 10^9$; $I = 1mA$; $U = 289.2m$; $Q = e$)

Energy of a particle:

$$E = \sqrt{c^2 \cdot p^2 + E_0^2}$$

(*Example* : $E = 3.7\text{GeV}$; $p = 3699.999965\text{GeV}/c$; *electrons*)

(*Example* : $E = 7.062603\text{GeV}$; $p = 7\text{GeV}/c$; *protons*)

Kinetic energy of a particle beam:

$$E_{kin}[J] = 1.60217733 \cdot 10^{-19} \cdot E_{kin}[eV]$$

(*Example* : $E_{total} = 2.89\text{MJ}$; $n = 2.2 \cdot 10^{13}$; $E = 820\text{GeV}$)

$$E_{kin}[J] = E_{kin}[m^2kg/sec^2]$$

(*Example* : $E_{total} = 3.09\text{MJ}$; $n = 1$; $m = 2000\text{kg}$; $v = 200\text{km}/h$)

α , β and γ function:

$$\gamma = \frac{1 + \alpha^2}{\beta}; \quad \alpha = -\frac{\beta'_r}{2}$$

β function at the end of a straight section of the length L :

$$\beta_L = \beta_0 - 2L \cdot \alpha_0 + L^2 \cdot \frac{1 + \alpha_0^2}{\beta_0}$$

Change of position and angle because of a kick in a storage ring:

$$dz = \frac{1}{2} \alpha_k \sqrt{\beta_k \cdot \beta_z} \cdot \frac{\cos(\pi Q_z - 2\pi|\varphi_z - \varphi_k|)}{\sin(\pi Q_z)}$$

$$dz' = -\frac{1}{2} \alpha_k \frac{\sqrt{\frac{\beta_k}{\beta_z}}}{\sin(\pi Q_z)} \cdot (\alpha_z \cdot \cos(\pi Q_z - 2\pi|\varphi_z - \varphi_k|) - \frac{\varphi_z - \varphi_k}{|\varphi_z - \varphi_k|} \cdot \sin(\pi Q_z - 2\pi|\varphi_z - \varphi_k|))$$

Change of position and angle because of a local bump kick:

$$dz = \alpha_k \sqrt{\beta_k \cdot \beta_z} \cdot \sin(\varphi_z - \varphi_k)$$

$$dz' = \alpha_k \sqrt{\frac{\beta_k}{\beta_z}} \cdot (\cos(\varphi_z - \varphi_k) - (-\frac{\beta'_z}{2}) \cdot \sin(\varphi_z - \varphi_k))$$

(*Example* : $dz = 2\text{mm}$; $\alpha_k = 0.1\text{mrad}$; $\beta_k = \beta_z = 20\text{m}/\text{rad}$; $\varphi_z - \varphi_k = \pi/2$)

Orbit lengthening because of a horizontal kick in a storage ring:

$$dl = D_x \cdot \alpha_k$$

Enlargement of the quadrupole strength in F(D) quadrupoles leads there to a reduction of the $\beta_x(\beta_z)$ function

Tune shift due to a change of the quadrupole strength:

$$\Delta Q = \frac{1}{4\pi} \cdot \beta \cdot L \cdot \Delta k$$

(*Example* : $\Delta Q = 0.1114$; $\beta = 20m$; $L = 0.7m$; $\Delta k = 0.1m^{-2}$)

Vertical tune shift due to a wiggler:

$$\Delta Q_z = \frac{\pi \cdot \beta_z \cdot N \cdot k^2}{2 \cdot \gamma^2 \cdot \lambda} = \frac{c^2 \cdot e^2 \cdot \beta_z \cdot L \cdot B_{max}^2}{8\pi \cdot E^2}$$

$$\Delta Q_z = 0.003576033 \cdot \beta_z[m] \cdot L[m] \cdot \frac{B_{max}[T]^2}{E[GeV]^2}$$

(*Example* : $\Delta Q_z = 0.0017$; $\beta_z = 13m$; $L = 2.112m$; $B_{max} = 0.60T$;
 $E = 4.5GeV$)

(*Example* : $\Delta Q_z = 0.0106$; $\beta_z = 16m$; $L = 2.400m$; $B_{max} = 1.25T$;
 $E = 4.5GeV$)

(*Example* : $\Delta Q_z = 0.0170$; $\beta_z = 24m$; $L = 4.000m$; $B_{max} = 1.00T$;
 $E = 4.5GeV$)

Fraction of the tune of a storage ring:

$$frac(Q) = \frac{1}{2\pi} \arccos\left(\frac{1}{2}trace(M)\right) = \frac{1}{2\pi} \arccos\left(\frac{1}{2}(M_{11} + M_{22})\right)$$

Tune of a storage ring:

$$Q = \frac{1}{2\pi} \cdot \int \frac{1}{\beta} ds$$

Chromaticity of a storage ring:

$$\frac{\Delta Q}{\frac{\Delta E}{E}} = \frac{1}{4\pi} \int (k - m \cdot D) \cdot \beta \cdot dl$$

($k < 0$: *F* - quadrupole, $m < 0$: *F* - sextupole)

Momentum compaction factor of a storage ring:

$$\alpha_c = \frac{\frac{\Delta L}{L}}{\frac{\Delta p}{p}} = \frac{\frac{\Delta T}{T}}{\frac{\Delta E}{E}} \text{ (ultra relativistic)}$$

$$\alpha_c = \frac{\frac{\Delta T}{T}}{\frac{\Delta p}{p}} + \frac{1}{\gamma^2} \text{ (relativistic)}$$

Transition energy:

$$\gamma_{tran} = \frac{E}{E_0} = \frac{1}{\sqrt{\alpha_c}}$$

(Example : $E = 3.9\text{MeV}$; electrons; $\alpha_c = 0.017$)

(Example : $E = 8.77\text{GeV}$; protons; $\alpha_c = 0.01145$)

Emittance after adiabatic damping in a storage ring:

$$\epsilon_x \sim \frac{1}{\beta \cdot \gamma} \text{ (relativistic)}$$

$$\epsilon_x \sim \frac{1}{\gamma} = \frac{E_0}{E} \text{ (ultra relativistic)}$$

Emittance of a radiation damped beam in a storage ring:

$$\epsilon_x \sim E^2$$

Emittance dependence in a storage ring:

$$\Delta\epsilon_x \sim \frac{D_x^2}{\beta_x \cdot abs(r)}, \text{ if } D'_x = 0, \alpha_x = 0 \text{ and } D_x \ll abs(r)$$

Beam dimensions in an electron storage ring:

$$\sigma_x = \sqrt{\epsilon_x \cdot \beta_x + (D_x \cdot \sigma_\epsilon)^2}$$

$$\sigma_z = \sqrt{\epsilon_z \cdot \beta_z}$$

$$\sigma'_x = \sqrt{\epsilon_x \frac{1 + \alpha_x^2}{\beta_x} + (D'_x \cdot \sigma_\epsilon)^2}$$

$$\sigma'_z = \sqrt{\epsilon_z \frac{1 + \alpha_z^2}{\beta_z}}$$

(Example : $\sigma_x = 3.0\text{mm}$; $\epsilon_x = 0.4\text{mm} \cdot \text{mrad}$; $\beta_x = 20\text{m/rad}$;

$D_x = 1\text{m}$; $\sigma_\epsilon = 0.001$)

(Example : $\sigma'_z = 28\mu\text{rad}$; $\epsilon_z = 0.008\text{mm} \cdot \text{mrad}$; $\beta_z = 10\text{m/rad}$; $\alpha_z = 0$)

Touschek scattering means elastic electron electron scattering within a bunch

Beam lifetime due to inelastic electron scattering with residual gas nucleons (bremsstrahlung production):

$$\tau_B = \frac{1}{\alpha \cdot c \cdot \frac{(4r_e)^2}{3} \cdot Z(Z+1) \cdot \ln(183 \cdot Z^{-1/3}) \cdot (-\ln \epsilon - \frac{5}{8}) \cdot N_g}$$

$$\tau_B[sec] = \frac{1.079323 \cdot 10^{22}}{Z(Z+1) \cdot \ln(183 \cdot Z^{-1/3}) \cdot (-\ln \epsilon - \frac{5}{8}) \cdot N_g[m^{-3}]}$$

$$N_g[m^{-3}] = \frac{\rho[kg/m^3]}{2Z \cdot m_p[kg]} \cdot \frac{p[mbar]}{1013}$$

$$\tau_B[sec] = \frac{0.0008019}{(-\ln \epsilon - \frac{5}{8}) \cdot p[mbar]} \quad \text{for } Z = 7$$

(Example : $\tau_B = 29890sec = 8.30h$; $Z = 7(\text{nitrogen})$; $\epsilon = 0.0025$;
 $N_g = 2.6347681 \cdot 10^{14}m^{-3}$; $\rho = 1.25kg/m^3$; $p = 5 \cdot 10^{-9}mbar$)

Beam lifetime due to elastic electron scattering with residual gas nucleons:

$$\tau_{el} = \frac{2E^2}{\pi \cdot c \cdot E_0^2 \cdot (2r_e \cdot Z)^2 \cdot (\frac{\langle \beta_x \rangle}{A_x} + \frac{\langle \beta_z \rangle}{A_z}) \cdot N_g}$$

$$\tau_{el}[sec] = \frac{2.5603272 \cdot 10^{26} \cdot E^2}{Z^2 \cdot (\frac{\langle \beta_x \rangle}{A_x} + \frac{\langle \beta_z \rangle}{A_z}) \cdot N_g[m^{-3}]}$$

$$N_g[m^{-3}] = \frac{\rho[kg/m^3]}{2Z \cdot m_p[kg]} \cdot \frac{p[mbar]}{1013}$$

$$\tau_{el}[sec] = \frac{99.16 \cdot E^2}{(\frac{\langle \beta_x \rangle}{A_x} + \frac{\langle \beta_z \rangle}{A_z}) \cdot p[mbar]} \quad \text{for } Z = 7$$

(Example : $\tau_{el} = 112070sec = 31.13h$; $Z = 7(\text{nitrogen})$; $\beta_x = 20m/rad$;
 $A_x = 80 \cdot 10^{-6}m$; $\beta_z = 20m/rad$; $A_z = 6 \cdot 10^{-6}m$;
 $N_g = 2.6347681 \cdot 10^{14}m^{-3}$; $\rho = 1.25kg/m^3$; $p = 5 \cdot 10^9mbar$)

Cavity power:

$$P = \frac{U^2}{2R_s}$$

(Example : $P = 110kW$; $U = 1.755MV$; $R_s = 14M\Omega$)

Dispersion in a cavity leads to a displacement of the closed orbit during acceleration or deceleration and therefore to an excitation of radial betatron oscillations

Polarization time in a storage ring (Sokolov-Ternov):

$$P_{max} = \frac{8}{5\sqrt{3}} = 0.9237604$$

$$\tau = \frac{8}{5\sqrt{3}} \cdot \frac{E_0^6 \cdot R \cdot r^2}{\hbar^2 \cdot c^2 \cdot r_e \cdot E^5}$$

$$\tau[sec] = 98.65992 \cdot \frac{R[m] \cdot r[m]^2}{E[GeV]^5} \quad \text{for electrons}$$

(*Example* : $\tau = 365\text{sec}$; $R = 46.0276\text{m}$; $r = 12.1849\text{m}$; $E = 4.5\text{GeV}$)

(*Example* : $\tau = 1513\text{sec}$; $R = 1008\text{m}$; $r = 608\text{m}$; $E = 30\text{GeV}$)

Luminosity:

$$L = \frac{1}{4\pi \cdot e^2 \cdot f_u} \cdot \frac{I_1 \cdot I_2}{\sigma_x \cdot \sigma_z}$$

(*Example* : $L = 2.49 \cdot 10^{28}\text{cm}^{-2}\text{sec}^{-1}$; $I_1 = 1\text{mA}$; $I_2 = 1\text{mA}$;

$f_u = 1.037\text{MHz}$; $\sigma_x = 0.075\text{cm}$; $\sigma_z = 0.0016\text{cm}$; ($\kappa = 1\%$))

Vertical tune shift at one interaction point (IP) in a storage ring (tune splitting; only the higher tune moves):

$$\Delta Q_z = \frac{r_e \cdot E_0 \cdot N_{bunch} \cdot \beta_z(IP)}{2\pi \cdot E \cdot \epsilon_x \cdot (\sqrt{\kappa \cdot \beta_x(IP) \cdot \beta_z(IP)} + \kappa \cdot \beta_z(IP))}$$

$$\Delta Q_z \approx \frac{r_e \cdot E_0 \cdot N_{bunch}}{2\pi \cdot E \cdot \sqrt{\epsilon_x \cdot \epsilon_z}} \cdot \sqrt{\frac{\beta_z(IP)}{\beta_x(IP)}}$$

(*Example* : $\Delta Q_z = 0.038$; $N_{bunch} = 2.5 \cdot 10^{11}$; $\beta_z(IP) = .09\text{m}$;

$E = 17\text{GeV}$; $\epsilon_x = 2 \cdot 10^{-7}\text{m}$; $\kappa = .013$; $\beta_x(IP) = 1.3\text{m}$)

(*Example* : $\Delta Q_z = 0.022$; $N_{bunch} = 2.7 \cdot 10^{11}$; $\beta_z(IP) = .04\text{m}$;

$E = 5.3\text{GeV}$; $\epsilon_x = 0.56 \cdot 10^{-6}\text{m}$; $\kappa = .05$; $\beta_x(IP) = .63\text{m}$)

(*Example* : $\Delta Q_z = 0.040$; $N_{bunch} = 2.5 \cdot 10^{10}$; $\beta_z(IP) = .04\text{m}$;

$E = 2\text{GeV}$; $\epsilon_x = 8.9 \cdot 10^{-8}\text{m}$; $\kappa = .03$; $\beta_x(IP) = .8\text{m}$)

Synchrotron frequency:

$$f_s = f_u \cdot \sqrt{\frac{H \cdot \alpha_c \cdot e \cdot U_{HF} \cdot \cos \psi_s}{2\pi \cdot E}}$$

(*Example* : $f_s = 39.9\text{kHz}$; $f_u = 1.037\text{MHz}$; $H = 482$; $\alpha_c = 0.016$;

$U_{HF} = 6\text{MV}$; $\psi_s = 20^\circ$; $E = 4.5\text{GeV}$)

Radiated power of a particle with charge q:

$$P = \frac{c \cdot q^2 \cdot E^4}{6\pi \cdot \epsilon_0 \cdot E_0^4 \cdot r^2}$$

$$P[W] = 6.762567 \cdot 10^{-7} \frac{E[\text{GeV}]^4}{r[\text{m}]^2} \text{ for electrons}$$

(*Example* : $P = 0.35\text{mW}$; $E = 50\text{GeV}$; $r = 110\text{m}$)

Radiated energy of an electron (per turn):

$$E_{ph} = \frac{e^2 \cdot E^4}{3\epsilon_0 \cdot E_0^4 \cdot r} = \frac{4\pi}{3} \cdot \frac{r_e}{E_0^3} \cdot \frac{E^4}{r}$$

$$E_{ph}[keV] = 88.46270 \cdot \frac{E[GeV]^4}{r[m]} \quad \text{for electrons}$$

$$(Example : E_{ph} = 2.977MeV ; E = 4.5GeV ; r = 12.1849m)$$

$$(Example : E_{ph} = 5.729MeV ; E = 5.3GeV ; r = 12.1849m)$$

$$(Example : E_{ph} = 5.03GeV ; E = 50GeV ; r = 110m)$$

Radiated synchrotron radiation power in a storage ring:

$$P = \frac{E_{ph}}{e} \cdot I ; \quad I = \frac{N_e \cdot e \cdot c}{U}$$

$$P = \frac{e}{3\epsilon_0 \cdot E_0^4} \cdot \frac{E^4}{r} \cdot I = \frac{4\pi \cdot r_e}{3e \cdot E_0^3} \cdot \frac{E^4}{r} \cdot I$$

$$P[kW] = 88.46270 \cdot \frac{E[GeV]^4}{r[m]} \cdot I[A] \quad \text{for electrons}$$

$$(Example : P = 297.7kW ; E = 4.5GeV ; r = 12.1849m ; I = 0.1A)$$

$$(Example : P = 572.9kW ; E = 5.3GeV ; r = 12.1849m ; I = 0.1A)$$

$$(Example : P = 574.0kW ; E = 12GeV ; r = 191.7295m ; I = 0.06A)$$

Radiated power of a bending magnet:

$$P = \frac{2 \cdot r_e}{3e \cdot E_0^3} \cdot L \cdot I \cdot \frac{E^4}{r^2} = \frac{2 \cdot c^2 \cdot r_e \cdot e}{3E_0^3} \cdot L \cdot I \cdot E^2 \cdot B^2$$

$$P[kW] = 14.07928 \cdot L[m] \cdot I[A] \cdot \frac{E[GeV]^4}{r[m]^2}$$

$$= 1.265382 \cdot L[m] \cdot I[A] \cdot E[GeV]^2 \cdot B[T]^2 \quad \text{for electrons}$$

$$(Example : P = 23.87kW ; L = 3.19m ; I = 0.1A ;$$

$$E = 5.3GeV ; r = 12.1849m)$$

Radiated power of a wiggler or undulator with sinusoidal field:

$$P[kW] = \frac{1.265382}{2} L[m] \cdot I[A] \cdot E[GeV]^2 \cdot B_{max}[T]^2 \quad \text{for electrons}$$

$$(Example : P = 0.659kW ; L = 2.112m ; I = 0.1A ;$$

$$E = 3.7GeV ; B_{max} = 0.60T)$$

$$(Example : P = 5.228kW ; L = 4.000m ; I = 0.1A ;$$

$$E = 4.5GeV ; B_{max} = 1.01T)$$

$$(Example : P = 1.696kW ; L = 2.4m ; I = 0.045A ;$$

$$E = 5.3GeV ; B_{max} = 0.94T)$$

$$(Example : P = 6.614kW ; L = 4.000m ; I = 0.06A ;$$

$$E = 12GeV ; B_{max} = 0.55T)$$

Central power density of a bending magnet for $\epsilon = 0$:

$$P = \frac{1}{0.608} \cdot \frac{2 \cdot r_e}{3 \cdot \sqrt{2\pi} \cdot e \cdot E_0^4} \cdot \frac{E^5}{r} \cdot I$$

$$P[W/mrad^2] = 18.1 \cdot \frac{E[GeV]^5}{r[m]} \cdot I[A] \text{ for electrons}$$

(*Example* : $P = 0.274kW/mrad^2$; $E = 4.5GeV$; $r = 12.1849m$; $I = 0.1A$)

Central power density of a wiggler or undulator for $\epsilon = 0$:

$$P[W/mrad^2] = 10.85 \cdot N \cdot E[GeV]^4 \cdot B_{max}[T] \cdot I[A] \text{ for electrons}$$

(*Example* : $P = 45.2kW/mrad^2$; $N = 127$; $E = 4.5GeV$;
 $B_{max} = 0.8T$; $I = 0.1A$)

(*Example* : $P = 94.3kW/mrad^2$; $N = 127$; $E = 12GeV$;
 $B_{max} = 0.55T$; $I = 0.06A$)

Critical wave length of the synchrotron radiation:

$$\lambda_c = \frac{4\pi}{3} \cdot \frac{E_0^3}{c \cdot B \cdot E^2} = \frac{4\pi}{3} \cdot \frac{r \cdot E_0^3}{E^3}$$

$$\lambda_c[\text{\AA}] = \frac{18.64353}{B[T] \cdot E[GeV]^2} = 5.589191 \cdot \frac{r[m]}{E[GeV]^3} \text{ for electrons}$$

(*Example* : $\lambda_c = 0.747\text{\AA}$; $r = 12.1849m$; $E = 4.5GeV$)

(*Example* : $\lambda_c = 0.457\text{\AA}$; $r = 12.1849m$; $E = 5.3GeV$)

(*Example* : $\lambda_c = 0.1616\text{\AA}$; $r = 608m$; $E = 27.6GeV$)

Critical energy of the synchrotron radiation:

$$E_c = \frac{3}{2} \cdot \frac{\hbar \cdot c}{E_0^3} \cdot \frac{E^3}{r} = \frac{2\pi \cdot \hbar \cdot c}{\lambda_c}$$

$$E_c[eV] = 2218.286 \cdot \frac{E[GeV]^3}{r[m]} = \frac{12398.42}{\lambda_c[\text{\AA}]} \text{ for electrons}$$

(*Example* : $E_c = 16589eV$; $E = 4.5GeV$; $r = 12.1849m$)

(*Example* : $E_c = 27103eV$; $E = 5.3GeV$; $r = 12.1849m$)

(*Example* : $E_c = 19993eV$; $E = 12GeV$; $r = 191.7295m$)

(*Example* : $E_c = 76708eV$; $E = 27.6GeV$; $r = 608m$)

Fundamental wave length and energy of an undulator:

$$\lambda_{ph} = \frac{\lambda}{2\gamma^2} \cdot \left(1 + \frac{k^2}{2} + \gamma^2 \cdot \theta^2\right); \quad \gamma = \frac{E}{E_0}$$

θ = observation angle with respect to the undulator beam

$$E_{ph}[eV] = \frac{12398.42}{\lambda_{ph}[\text{\AA}]}$$

(Example : $\lambda_{ph} = 4.648\text{\AA}$; $\lambda = 3.14\text{cm}$; $E = 4.5\text{GeV}$; $k = 1.61$; $\theta = 0$)

(Example : $E_{ph} = 2667\text{eV}$; $\lambda_{ph} = 4.648\text{\AA}$)

(Example : $\lambda_{ph} = 0.6537\text{\AA}$; $\lambda = 3.14\text{cm}$; $E = 12\text{GeV}$; $k = 1.61$; $\theta = 0$)

(Example : $E_{ph} = 18967\text{eV}$; $\lambda_{ph} = 0.6537\text{\AA}$)

Undulator beam divergence for the i^{th} harmonics:

$$\sigma'_{ph} = \sqrt{\frac{1 + \frac{k^2}{2}}{2i \cdot N \cdot \gamma^2}}$$

(Example : $\sigma'_{ph} = 10.8\mu\text{rad}$; $k = 1.61$; $i = 1$; $N = 128$; $\gamma = 8806$)

(Example : $\sigma'_{ph} = 2.33\mu\text{rad}$; $k = 1.61$; $i = 3$; $N = 128$; $\gamma = 23483$)

Undulator beam divergence (fundamental mode; zero electron emittance):

$$\sigma'_{ph} = \sqrt{\frac{\lambda_u \cdot (1 + \frac{k^2}{2})}{2\gamma^2 \cdot L}}$$

$$\sigma'_{ph} = \sqrt{\frac{\lambda_{ph}}{L}}; \quad \lambda_{ph} = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{k^2}{2}\right)$$

(Example : $\sigma'_{ph} = 6.12\mu\text{rad}$; $\lambda_{ph} = 0.15\text{nm}$; $L = 4\text{m}$)

Undulator beam size for zero electron emittance:

$$\sigma_{ph} = \frac{1}{4\pi} \cdot \sqrt{\lambda_{ph} \cdot L}$$

(Example : $\sigma_{ph} = 31.5\mu\text{m}$; $\lambda_{ph} = 6.28\text{nm}$; $L = 25\text{m}$)

Photon beam emittance for zero electron emittance:

$$\epsilon_{ph} = \frac{\lambda_{ph}}{4\pi}$$

(Example : $\epsilon_{ph} = 0.5\text{nmrad}$; $\lambda_{ph} = 6.28\text{nm}$)

Photon beam coherence angle:

$$\sigma'_{coh} = \frac{\lambda_{ph}}{4\pi \cdot \sigma_{total}}; \quad \sigma_{total} = \sqrt{\sigma_e^2 + \sigma_{ph}^2}$$

(*Example* : $\sigma'_{coh} = 362nrad$; $\lambda_{ph} = 0.15nm$; $\sigma_{total} = 33\mu m$)

Undulator line width for the N^{th} harmonics and for n periods:

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{n \cdot N}$$

(*Example* : $\Delta\lambda/\lambda = 0.26\%$; $n = 128$; $N = 3$)

Undulator line broadening because of the electron beam divergence:

$$\frac{\Delta\lambda}{\lambda} = \frac{(\gamma \cdot \sigma'_r)^2}{1 + \frac{k^2}{2}}$$

(*Example* : $\Delta\lambda/\lambda = 8.8\%$; $E = 4.5GeV$; $\sigma'_r = 51\mu rad$; $k = 1.61$)

Undulator line broadening because of the acceptance angle θ :

$$\frac{\Delta\lambda}{\lambda} = \frac{(\gamma \cdot \theta)^2}{1 + \frac{k^2}{2}}$$

Undulator line broadening because of the electron energy spread:

$$\frac{\Delta\lambda}{\lambda} = 2 \frac{\Delta E}{E}$$

(*Example* : $\Delta\lambda/\lambda = 0.22\%$; $\Delta E/E = 0.11\%$)

Divergence of synchrotron radiation from a bending magnet (vertical plane):

$$\theta = 0.608 \cdot \frac{E_0}{E} = \frac{0.608}{\gamma}$$

$$\theta = \frac{E_0}{E} \cdot \left(\frac{\lambda}{\lambda_c}\right)^{1/3} \text{ for } \lambda \gg \lambda_c; \quad \lambda > 507 \cdot \lambda_c$$

$$\theta = 0.565 \cdot \frac{E_0}{E} \cdot \left(\frac{\lambda}{\lambda_c}\right)^{0.425} \text{ for } \lambda > 0.750 \cdot \lambda_c$$

$$\theta = \frac{E_0}{E} \cdot \left(\frac{\lambda}{3\lambda_c}\right)^{1/2} \text{ for } \lambda < 0.750 \cdot \lambda_c$$

(*Example* : $\theta = 0.069mrad$; $E = 4.5GeV$)

(*Example* : $\theta = 0.514mrad$; $E = 4.5GeV$; $\lambda = 100\text{\AA}$; $\lambda_c = 0.747\text{\AA}$)

(*Example* : $\theta = 2.209mrad$; $E = 4.5GeV$; $\lambda = 5500\text{\AA}$; $\lambda_c = 0.747\text{\AA}$)

Divergence of synchrotron radiation from a bending magnet (horizontal plane):

$$\theta = \frac{7}{12} \cdot \frac{0.608}{\gamma}$$

Spectral flux (bending magnet):

$$I[\text{phot.}/(\text{sec mrad } 0.1\% \text{ bandw.})] = 2.458 \cdot 10^{10} \cdot I_e[\text{mA}] \cdot E_e[\text{GeV}] \cdot \frac{E}{E_c} \cdot \int_{E/E_c}^{\infty} K_{5/3}(\eta) d\eta$$

$$\int_1^{\infty} K_{5/3}(\eta) d\eta = 0.6522$$

(*Example* : $I = 7.214 \cdot 10^{12} \text{phot.}/(\text{sec mrad } 0.1\% \text{ bandw.})$;
 $I_e = 100 \text{mA}$; $E_e = 4.5 \text{GeV}$; $E = E_c$)

Spectral flux (wiggler):

$$I[\text{phot.}/(\text{sec mrad } 0.1\% \text{ bandw.})] = 2.458 \cdot 10^{10} \cdot 2N \cdot I_e[\text{mA}] \cdot E_e[\text{GeV}] \cdot \frac{E}{E_c} \cdot \int_{E/E_c}^{\infty} K_{5/3}(\eta) d\eta$$

(*Example* : $I = 2.02 \cdot 10^{14} \text{phot.}/(\text{sec mrad } 0.1\% \text{ bandw.})$;
 $N = 28$; $I_e = 100 \text{mA}$; $E_e = 4.5 \text{GeV}$; $E = E_c$)

Spectral central brightness for $\epsilon = 0$ (bending magnet):

$$I[\text{phot.}/(\text{sec mrad}^2 \text{ } 0.1\% \text{ bandw.})] = 1.325 \cdot 10^{10} \cdot I_e[\text{mA}] \cdot E_e[\text{GeV}]^2 \cdot \left(\frac{E}{E_c}\right)^2 \cdot K_{2/3}\left(\frac{E}{2E_c}\right)$$

$$K_{2/3}\left(\frac{1}{2}\right) \approx 1.45$$

(*Example* : $I \approx 3.9 \cdot 10^{13} \text{phot.}/(\text{sec mrad}^2 \text{ } 0.1\% \text{ bandw.})$;
 $I_e = 100 \text{mA}$; $E_e = 4.5 \text{GeV}$; $E = E_c$)

Spectral central brightness for $\epsilon = 0$ (wiggler):

$$I[\text{phot.}/(\text{sec mrad}^2 \text{ } 0.1\% \text{ bandw.})] = 1.325 \cdot 10^{10} \cdot 2N \cdot I_e[\text{mA}] \cdot E_e[\text{GeV}]^2 \cdot \left(\frac{E}{E_c}\right)^2 \cdot K_{2/3}\left(\frac{E}{2E_c}\right)$$

(*Example* : $I \approx 2.2 \cdot 10^{15} \text{phot.}/(\text{sec mrad}^2 \text{ } 0.1\% \text{ bandw.})$;
 $N = 28$; $I_e = 100 \text{mA}$; $E_e = 4.5 \text{GeV}$; $E = E_c$)

SASE FEL ρ parameter (1 dimensional theory):

$$\rho = \left(\frac{k^2 \cdot r_e \cdot n_b \cdot \lambda_u^2 \cdot J J^2(k)}{32\pi \cdot \gamma^3} \right)^{1/3}$$

$$n_b = \frac{n}{\sqrt{2\pi}^3 \cdot \sigma_r^2 \cdot \sigma_s}$$

$$\frac{\epsilon_n}{\gamma} < \frac{\lambda_{\text{phot}}}{4\pi}$$

$$\sigma_e < \frac{\rho}{2}$$

(*Example* : $\rho = 0.00222$; $k = 1.266$; $n_b = 3.17 \cdot 10^{21} m^{-3}$;
 $\lambda_u = 2.73 cm$; $JJ(k) = 0.877$; $E = 1 GeV$; $n = 6.24 \cdot 10^9$;
 $\sigma_r = 50 \mu m$; $\sigma_s = 50 \mu m$; $\epsilon_n = 1 \pi mm \cdot mrad$;
 $\lambda_{phot} = 6.24 nm$; $\sigma_e = 0.001$)

Saturation length of SASE FEL (1 dimensional theory):

$$L_s = \lambda_u / \rho$$

(*Example* : $L_s = 12.3 m$; $\lambda_u = 2.73 cm$; $\rho = 0.00222$)

Photon peak power of SASE FEL (1 dimensional theory):

$$P_e = \frac{c \cdot n \cdot E_e}{\sqrt{2\pi} \cdot \sigma_s}$$

(*Example* : $P_e = 2.4 TW$; $n = 6.24 \cdot 10^9$; $E_e = 1 GeV$; $\sigma_s = 50 \mu m$)

$$P_{phot} = \rho \cdot P_e$$

(*Example* : $P_{phot} = 5.3 GW$; $\rho = 0.00222$; $P_e = 2.4 TW$)

Speed of light:

$$c = 2.99792458 \cdot 10^8 m/sec$$

Rest energy / rest mass:

$$E_0(\text{electrons}) = 0.51099906 MeV; \quad m_e = 9.1093897 \cdot 10^{-31} kg$$

$$E_0(\text{protons}) = 938.27231 MeV; \quad m_p = 1.6726231 \cdot 10^{-27} kg$$

$$1[kg] = 1[V A sec^3 / m^2]$$

Classical electron radius:

$$r_e = 2.81794092 \cdot 10^{-15} m; \quad r_e = \frac{e^2}{4\pi \cdot \epsilon_0 \cdot E_0}$$

Electron charge magnitude:

$$e = 1.60217733 \cdot 10^{-19} Asec; \quad 1[eV] = 1.60217733 \cdot 10^{-19} [Wsec]$$

Permeability of free space:

$$\mu_0 = 4\pi \cdot 10^{-7} Vsec/Am$$

Permittivity of free space:

$$\epsilon_0 = 8.854187817 \cdot 10^{-12} \text{As ec/Vm}$$

$$\epsilon_0 \cdot \mu_0 \cdot c^2 = 1$$

Planck constant:

$$\hbar = 6.5821220 \cdot 10^{-22} \text{MeV sec} = 1.05457266 \cdot 10^{-34} \text{W sec}$$

Fine structure constant:

$$\alpha = \frac{1}{137.0359895}; \quad \alpha = \frac{e^2}{4\pi \cdot \epsilon_0 \cdot \hbar \cdot c}$$

Gaussian curve:

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$$

$$f_{max}(x) = \frac{1}{\sqrt{2\pi}}$$

$$\int_{-0.5 \cdot \sqrt{2\pi}}^{0.5 \cdot \sqrt{2\pi}} f_{max} dx = 1$$

$$\sqrt{2\pi} = 2.506628$$

$$FWHM = 2\sqrt{2\ln 2} = 2.354820$$

$$\int_{-1}^1 f(x) dx = 0.6827$$

$$\int_{-0.5 \cdot FWHM}^{0.5 \cdot FWHM} f(x) dx = 0.761$$

$$\int_{-2}^2 f(x) dx = 0.9545$$

$$\int_{-3}^3 f(x) dx = 0.9973$$